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in one direction. This is possible if a large number of insulating barriers are so arranged that the average depth of the electrolyte between the strips is much greater than the average depth over their surface. A section through the strips is shown in Fig. 1 and a photograph of a rectangular model illustrated in Fig. 2. There are 48 strips 1 in. wide and $\frac{1}{4}$ in. thick each spaced $\frac{1}{4}$ in. apart and surrounded by a silver plated electrode 2×1 ft. The center electrode is a piece of rod $\frac{1}{8}$ in. in diameter. This model represents the distribution of current in a rectangular packet of laminations which have a relatively low interlaminar resistance. Figure 3 shows the distribution in a homogeneous medium, and Fig. 4 illustrates the effects of anisotropy. It is interesting to note how the equipotential lines are moved toward the outside laminations with increasing resistivity ratio.

This technique is now being used to obtain the flux distribution in transformer cores of grain-oriented steel.

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Low-Temperature Thermal Noise Thermometer*

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The design for a relatively simple noise thermometer for use at low temperatures is presented. This thermometer has been used to measure temperatures as low as the lambda point of liquid helium with a least accuracy of 10%. Some of the properties of thermal noise are discussed.

INTRODUCTION

THERMAL electrical noise has been the subject of extensive experimental and theoretical research beginning with the pioneering work of Johnson[1] and Nyquist.[2] One of the prime objectives of most of this work, aside from that of gaining a better understanding of thermal noise itself, has been to learn how to minimize where possible the effects of thermal noise. Even though thermal noise is generally a nuisance there are a few instances where it may serve as a source of useful information. Johnson[1] took advantage of thermal noise in making a measurement of Boltzmann's constant, much later, Garrison and Lawson[3] demonstrated that thermal noise provides a basis for an accurate thermometer of extremely wide range. It is the purpose of the present work to review some of the properties of thermal noise and to describe a relatively simple thermal noise thermometer which is useful as low as liquid helium temperatures.

THEORY

Nyquist's theorem states that the mean square thermal noise voltage across a resistance R between the frequencies f and $f+df$ is given by

$$\langle v^2(f)\rangle = 4kTR(f)df, \tag{1}$$

where k is Boltzmann's constant, T is the absolute temperature, and $R(f)$ is the resistance at the frequency f. This equation was derived by a classical thermodynamic argument for which the result is independent of the mechanism of thermal noise production. A slight refinement of the treatment of Nyquist which takes into account quantum effects leads to the result that

$$\langle v^2(f)\rangle = 4R(f)hf\left(\exp\frac{hf}{kT}-1\right)^{-1}df. \tag{2}$$

Here, h is Planck's constant and the other terms have their usual meanings. For the ordinary environment where $hf/kT \ll 1$, Eq. (2) reduces to Eq. (1) and hence Nyquist's original equation holds. As pointed out by van der Ziel,[4] the difference in the predictions of Eqs. (1) and (2) under conditions where $hf/kT \approx 1$ appears to be barely detectable by employing present day techniques. This would require using frequencies of the order of 3×10^{10} cps and temperatures of the order of 0.02° K.

A high-gain amplifier must be used to observe thermal noise experimentally. For example, the rms noise signal in a 10^4-ohm resistor at a uniform temperature of 1° K as observed with a 20-kc band width is only 1.05×10^{-7} v. Once the noise resistance on which observations are to be made is connected to the amplifier, the noise circuit becomes everything which is connected to the grid of the first stage of amplification. In the simplest case, this means

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[1] J. B. Johnson, Phys. Rev. **32**, 97 (1928).
[2] H. Nyquist, Phys. Rev. **32**, 110 (1928).
[3] J. B. Garrison and A. W. Lawson, Rev. Sci. Instr. **20**, 785 (1949).
[4] A. van der Ziel, *Noise* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1954).

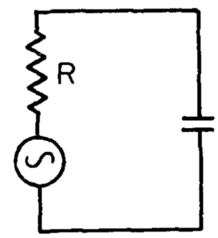

FIG. 1. Equivalent noise circuit at amplifier input.

that noise observations are being made on a resistance which is shunted by a capacitance.

The experiments of Williams[5] indicate that regardless of the nature of a passive electrical circuit, i.e., regardless of the combination of resistance, inductance, and capacitance, the thermal noise of the circuit is governed by the resistance temperature. The circuit is equivalent, as far as noise production is concerned, to one in which each resistance has a noise generator as described by Eq. (1) in series with it. The noise circuit at the input of the amplifier is equivalent to the circuit shown as Fig. 1. For the circuit of Fig. 1 elementary analysis shows that the mean square voltage across the capacitor between the frequencies f_1 and f_2 is given by

$$\langle V_c^2\rangle=\int_{f_1}^{f_2}\frac{4kTR(f)df}{4\pi^2f^2C^2R^2(f)+1}. \tag{3}$$

If the resistance is assumed to be independent of frequency, which is reasonable when f_2 is not taken too large, then,

$$\langle V_c^2\rangle=\frac{2kT}{\pi C}[\tan^{-1}2\pi RCf_2-\tan^{-1}2\pi RCf_1]. \tag{4}$$

Furthermore, if $2\pi RCf_2$ is required to be much less than one then the arc tangent may be replaced by its argument, with the result that

$$\langle V_c^2\rangle=4kTR(f_2-f_1), \tag{5}$$

which is the same expression which would be obtained for an unshunted resistance. For a resistance of 10^4 ohms, a capacitance of 50 $\mu\mu f$, and an upper frequency limit of 25 kc, the approximations made above introduce an error less than 0.5%.

The noise power output of the amplifier due to the noise signal from the resistance R is directly proportional to $\langle V_c^2\rangle$ and, as indicated by Eq. (5), this in turn is directly proportional to the magnitude of the resistance and the absolute temperature of the resistance. Consider a measured resistance R_1 which is kept at a uniform but unknown temperature T_1 and a variable resistance R_2 which is kept at a standard temperature T_2. If the noise signals in these resistances are measured using the same amplifying device under conditions such that Eq. (5) is valid and if the value of R_2 is varied until the observed amplifier output caused by this resistance is equal to that caused by the resistance R_1, then the temperature T_1 is related to the standard temperature T_2 by the simple equation

$$T_1=\frac{R_2}{R_1}T_2. \tag{6}$$

In making quantitative thermal noise measurements, a noise detector is required in addition to a high-gain amplifier. A complete discussion of this aspect of noise measurement has been given by van der Ziel.[4]

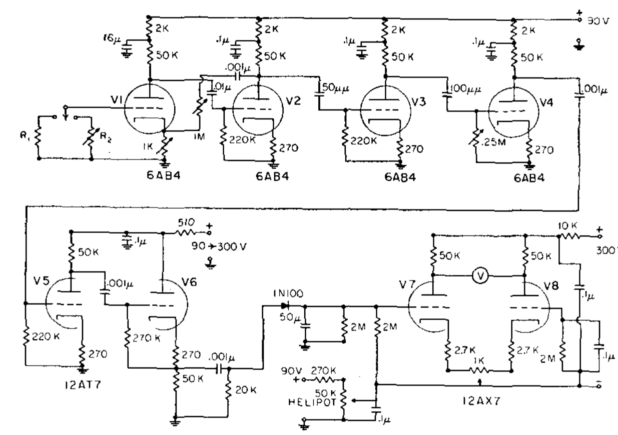

FIG. 2. Thermal noise thermometer circuit diagram.

CIRCUIT DESCRIPTION AND OPERATION

The sensing resistors are noninductive wire-wound units of either Constantan or Manganin wire ranging in values up to 10^4 ohms. The sensing resistors are connected by short lengths of low-capacity coaxial cable to a shielded switching box which is connected to the amplifier input. Switching between the sensing resistance and the variable resistance which is kept at the standard temperature is accomplished in this switching box which also serves as the housing for the variable resistance.

The noise amplifier and detector circuit is given in Fig. 2. Stages $V1$ through $V5$ constitute a narrow-band resistance-coupled amplifier, $V6$ is a cathode follower used to drive a crystal diode power detector, and $V7$ and $V8$ form a difference amplifier which drives an indicating meter. The entire system is powered by batteries. $V1$ and $V2$ are the most critical stages in the amplifier and are operated as a feedback pair. This feedback is adjusted to provide gain stability and at the same time aids in making the gain of this pair practically independent of input source impedance at least for values of sensing resistances up to 10^4 ohms. The coupling constants are chosen to make the amplifier responsive between 5 and 25 kc. The low-frequency limit of 5 kc is necessary to make the amplifier insensitive to flicker noise which is the predominant source of noise in

[5] F. C. Williams, J. Inst. Elec. Engrs. (London) 83, 76 (1938).

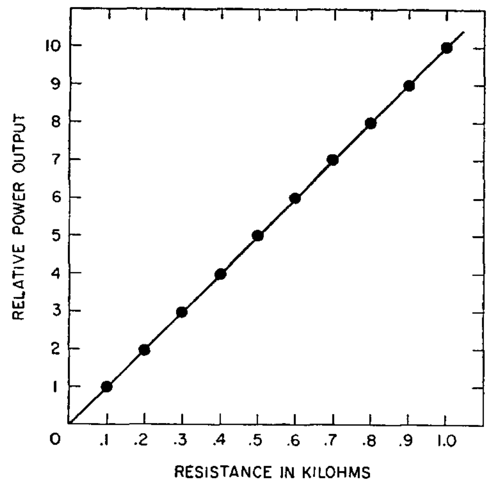

FIG. 3. Amplified noise power output *vs* sensing resistance for a temperature of 293°K.

vacuum electron tubes at lower frequencies. The noise detector output couples through a bias network to $V7$. In operation, the variable resistance at the amplifier input is set to zero and the bias network voltage is adjusted until the output meter reads zero. Under this adjustment, the output indication is zero when the signal from the detector is due to the amplifier noise alone. The input is then switched to the sensing resistance. The output indication is now a measure of the mean square noise voltage and hence the temperature of this resistance only since the amplifier noise and the thermal noise in the sensing resistance are uncorrelated. The input is then switched to the variable resistance and its value is adjusted until the output indication is the same as that obtained with the sensing resistance. The temperature of the sensing resistance is then given by Eq. (6).

The construction techniques employed in the noise thermometer are critical. Special care must be taken not to introduce additional noise sources. In particular it is important to use either wirewound or deposited film resistors for those which are required to conduct direct current. Individual shielding of each stage is recommended.

EXPERIMENTAL RESULTS AND CONCLUSIONS

Figure 3 illustrates the observed linear relationship between amplified noise power output and value of sensing resistance for a fixed sensing resistance temperature. Table I gives a typical set of temperature measurements made with the noise thermometer. The least accuracy of determining the lambda point of liquid helium on several successive occasions was found to be 10%.

TABLE I. Typical noise thermometer temperature measurement data. (Room temperature taken as reference T_2.)

Environment	Accepted temperature	Noise thermometer	Approximate error
Liquid nitrogen	77.3°K	75.7°K	2%
Liquid helium	4.2°K	4.4°K	5%
Liquid helium at lambda point	2.19°K	2.37°K	8%

The upper temperature limit of a noise thermometer such as the one described herein is determined only by the physical characteristics of the sensing resistance, i.e., melting point, etc. The low-temperature limit is set by the amplifying system itself and depends principally on the factors of gain, gain stability, and inherent amplifier noise. An amplifier of conventional design such as the one given here has an equivalent noise resistance of the order of 10^3 ohms. This amounts to an input noise level of 0.55 μv. The equivalent noise resistance of an amplifier is defined as that resistance which if connected at the input of a noiseless amplifier would produce the same noise output as occurs in the actual amplifier. In the present case, using a 10^4-ohm sensing resistance, the thermometer noise and the amplifier noise referred to the amplifier input are equal at about 30°K. The accuracy of the noise thermometer begins to decrease below this temperature.

A comparison of the noise thermometer with other types of thermometers which operate in the low-temperature range indicates that the noise thermometer has relatively poor accuracy. However, the noise thermometer does have the distinct advantage that it offers an absolute thermodynamic scale which need not be calibrated except at one fixed reference temperature.

It appears that it may be possible to extend the low-temperature limit of the thermometer to lie in the range between 0.1 and 1° K with a concomitant increase in accuracy at the higher temperatures. However, no radical improvement beyond that mentioned is foreseen without the advent of an amplifying device of lower inherent noise than ordinary triode electron tubes.

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